10.9

POLAR EQUATION OF CONIC
WITH FOCUS @ POLE &
DIRECTRIX p UNITS FROM FOCUS

	DIRECTRIX	VERTICAL	HORIZONTAL
+	RIGHT/ ABOVE	$r = \frac{ep}{1+e\cos\theta}$	$r = \frac{ep}{1+e\sin\theta}$
−	LEFT/ BELOW	$r = \frac{ep}{1-e\cos\theta}$	$r = \frac{ep}{1-e\sin\theta}$

$$\boxed{r = \frac{ep}{1 \pm e \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} \theta}}$$

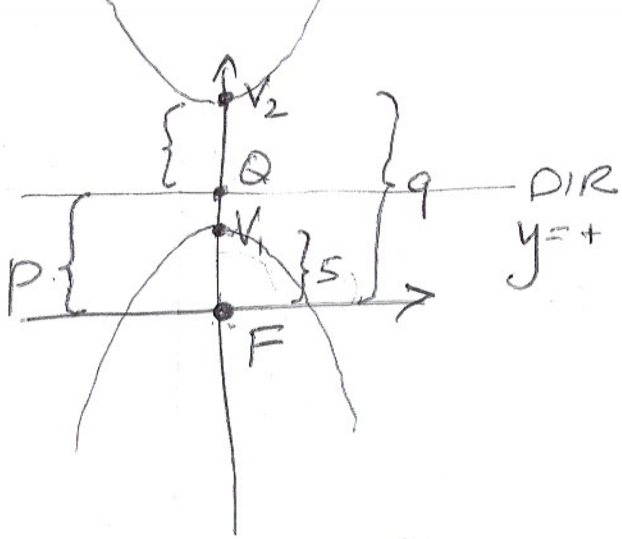
+x, +y
DIR IN POSITIVE TERRITORY ⟷ +
NEGATIVE ⟷ −
−x, −y

DIR IS
cos ⟷ VERTICAL $(x = p)$
sin ⟷ HORIZONTAL $(y = p)$

④ VERTICES $(x,y)=(0,5),(0,9)$

$$r=\frac{ep}{1+e\sin\theta}=\frac{\cancel{\frac{7}{2}}\cdot\frac{45}{\cancel{7}}}{1+\frac{7}{2}\sin\theta}\cdot\frac{2}{2}$$

$$r=\frac{45}{2+7\sin\theta}$$



$$e=\frac{V_1F}{V_1Q}=\frac{5}{p-5}$$

=

$$e=\frac{V_2F}{V_2Q}=\frac{9}{9-p}$$

$$\frac{5}{p-5}=\frac{9}{9-p}$$

$$5(9-p)=9(p-5)$$

$$45-5p=9p-45$$

$$90=14p$$

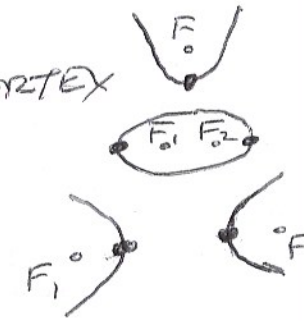
$$p=\frac{90}{14}=\frac{45}{7}$$

$$e=\frac{5}{\frac{45}{7}-5}\cdot\frac{7}{7}=\frac{35}{45-35}=\frac{35}{10}=\frac{7}{2}$$

PARABOLA HAS 1 VERTEX

ELLIPSE 2

HYPERBOLA 2



ELLIPSE → DIR OUTSIDE

HYPERBOLA → DIR BETWEEN BRANCHES

DIR IS NEVER AN AXIS OF SYM

↑

CAN BE PROVEN BY ALGEBRA

$$r = \frac{45}{2+7\sin\theta}$$

$$\underset{(x,y)}{(0,5)} \longrightarrow \underset{(r,\theta)}{\left(5, \frac{\pi}{2}\right)}$$

$$\frac{45}{2+7\sin\frac{\pi}{2}} = \frac{45}{2+7\cdot 1} = \frac{45}{9} = 5$$

$$\underset{(x,y)}{(0,9)} \longrightarrow \cancel{\left(9, \frac{\pi}{2}\right)} \text{ or } \left(-9, \frac{3\pi}{2}\right)$$

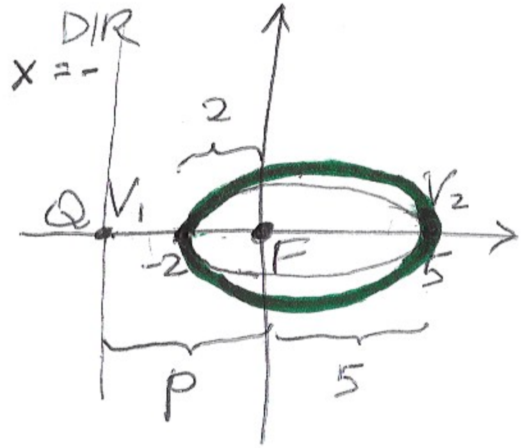
$$\frac{45}{2+7\sin\frac{3\pi}{2}} = \frac{45}{2+7(-1)} = \frac{45}{-5} = -9$$

FIND POLAR EQ'N OF CONIC WITH FOCUS @ POLE AND VERTICES $(x,y) = (-2,0)$ AND $(5,0)$

$$r = \frac{ep}{1 - e\cos\theta} = \frac{\cancel{\frac{3}{7}} \cdot \frac{20}{\cancel{3}}}{1 - \frac{3}{7}\cos\theta} \cdot \frac{7}{7}$$

$x = (-)$ $(x) =$

$$\boxed{r = \frac{20}{7 - 3\cos\theta}}$$



$$e = \frac{V_1F}{V_1Q} = \frac{2}{p-2}$$

$$e = \frac{V_2F}{V_2Q} = \frac{5}{p+5}$$

SANITY CHECK:

$(-2,0) \longrightarrow (r,\theta) = (\underline{\underline{2}},\pi)$ $\quad r = \frac{20}{7-3\cos\pi} = \frac{20}{7-3(-1)} = \frac{20}{10} = 2$ ✓

$(5,0) \longrightarrow (r,\theta) = (\underline{\underline{5}},0)$ $\quad r = \frac{20}{7-3\cos 0} = \frac{20}{7-3(1)} = \frac{20}{4} = 5$ ✓

ONLY USED TO DETECT OBVIOUS ERRORS

$\downarrow$

SANITY CHECKS

$p > 0$

e IS IN CORRECT INTERVAL

$$\frac{2}{p-2} = \frac{5}{p+5}$$

$$2p + 10 = 5p - 10$$

$$20 = 3p$$

$p = \frac{20}{3}$ $\longleftarrow$ p IS DISTANCE $\rightarrow p > 0$ ✓

$e = \frac{5}{\frac{20}{3}+5} \cdot \frac{3}{3} = \frac{15}{20+15} = \frac{15}{35} = \frac{3}{7}$ $\longleftarrow$ ELLIPSE $0 < e < 1$ ✓

GRAPH $r = \dfrac{4}{1-\sin\theta}$

① WRITE IN STANDARD FORM $r = \dfrac{ep}{1 \pm e \,{}^{\sin}_{\cos}\theta}$

② FIND e, p + EQ'N OF DIRECTRIX + SHAPE

$1 - 1\sin\theta$

$e = 1$

$\downarrow$

PARABOLA

$ep = 4$

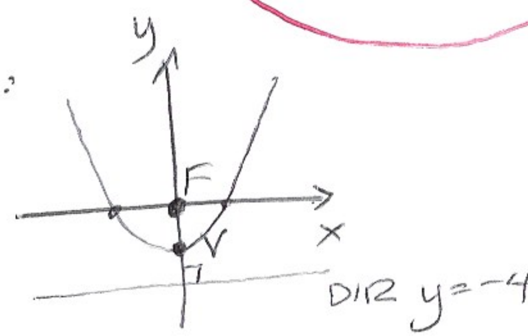
$1p = 4$

$p = \underline{4}$

$1 \underline{-} \underline{\sin}\theta$

$y = -4$

ROUGH SKETCH:

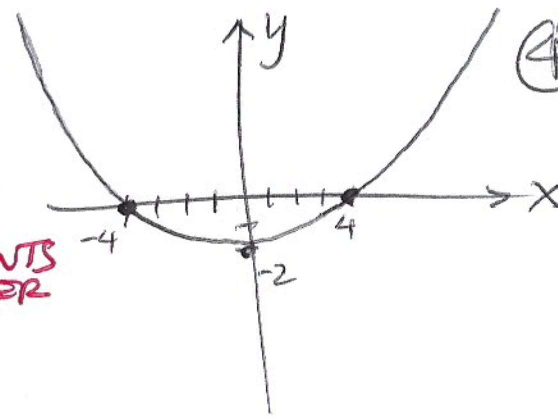


③ FIND x-, y- INTERCEPTS CORRESPONDING TO $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

	θ	r
($r>0$) POS X-AXIS	0	$\frac{4}{1-\sin 0} = \frac{4}{1-0} = 4$
	$\frac{\pi}{2}$	$\frac{4}{1-\sin\frac{\pi}{2}} = \frac{4}{1-1}$ DNE
NEG X-AXIS	π	$\frac{4}{1-\sin\pi} = \frac{4}{1-0} = 4$
NEG Y-AXIS	$\frac{3\pi}{2}$	$\frac{4}{1-\sin\frac{3\pi}{2}} = \frac{4}{1--1} = 2$

SAME ↓ POINTS SYM OVER POLE

④ PLOT + GRAPH



GRAPH $r = \dfrac{15}{4+\cos\theta} \cdot \dfrac{\frac{1}{4}}{\frac{1}{4}} = \dfrac{\boxed{\frac{15}{4}}}{1+\frac{1}{4}\cos\theta}$

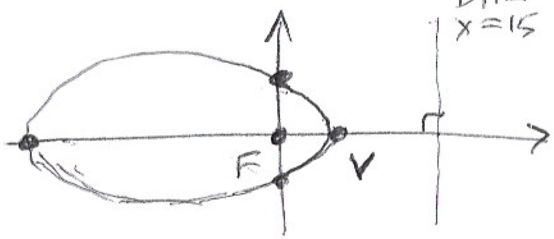
① WRITE IN STANDARD FORM $r = \dfrac{ep}{1 \pm e\cos\theta}$

② FIND e, p, DIR + SHAPE

$e = \frac{1}{4}$ $\quad ep = \frac{15}{4}$ $\quad$ DIR: $x = +15$

$\frac{1}{4}p = \frac{15}{4}$ $\quad$ SHAPE: $0 < e < 1 \longrightarrow$ ELLIPSE

$p = 15$



③ FIND X-, Y- INT

	θ	r
POS X-INT	0	$\frac{15}{4+\cos 0} = \frac{15}{4+1} = 3$
POS Y-INT	$\frac{\pi}{2}$	$\frac{15}{4+\cos\frac{\pi}{2}} = \frac{15}{4+0} = \frac{15}{4}$
NEG X-INT	π	$\frac{15}{4+\cos\pi} = \frac{15}{4+-1} = 5$
NEG Y-INT	$\frac{3\pi}{2}$	$\frac{15}{4+\cos\frac{3\pi}{2}} = \frac{15}{4+0} = \frac{15}{4}$

$r > 0$